

# Rossmoyne Senior High School

### Semester One Examination, 2017

### Question/Answer booklet

# MATHEMATICS

**APPLICATIONS**

**UNIT 3**

## Section One:

## Calculator-free

 Teacher name

 Your name

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Workingtime (minutes) | Marks available | Percentage of examination |
| Section One:Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:Calculator-assumed | 11 | 11 | 100 | 98 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**eight (****8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (4 marks)

**(a)** Write the recursive definition for the following sequences.

 **(i)** 400, 100, 25, ………….. (2 marks)

|  |
| --- |
| **Solution** |
| $$T\_{n+1}=0.25T\_{n} : T\_{1}=400$$ |
| **Specific behaviours** |
| ✓ States pattern✓ States first term |

 **(ii)** 4x, 8x, 12x, ………….. (2 marks)

|  |
| --- |
| **Solution** |
| $$T\_{n+1}=T\_{n}+4x : T\_{1}=4x$$ |
| **Specific behaviours** |
| ✓ States pattern✓ States first term |

Question 2 (5 marks)

Ten students were set a challenge in which they had to run through an obstacle course and solve problems at various locations along the way to score points. Their times, in minutes, and total scores are shown in the table below.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Time ($x$) | 34 | 33 | 24 | 26 | 30 | 28 | 31 | 30 | 23 | 25 |
| Score ($y$) | 19 | 19 | 31 | 30 | 24 | 27 | 21 | 28 | 33 | 34 |

(a) Construct a scatterplot on the axes below that can be used to identify whether any association exists between the variables. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ at least four points plotted correctly✓ at least eight points plotted correctly✓ all points plotted correctly |

(b) Describe the association between the two variables, $x$ and $y$. (2 marks)

|  |
| --- |
| **Solution** |
| Strong, negative, linear association |
| **Specific behaviours** |
| ✓ two of strength, direction, form✓ all three indicators |

**Question 3 (6 marks)**

**(a)** Determine the next three terms for the number sequence that is described

 using the following recursive equation. (3 marks)

 

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| --- |
| **Solution** |
| $$T\_{1}=4, T\_{2}=5 T\_{3}=8$$ |
| **Specific behaviours** |
| ✓ States $T\_{1}$✓ States $T\_{2}$✓ States $T\_{3}$ |

**(b)** The graph shown represents an arithmetic sequence.



**(i)** Explain why it is an arithmetic sequence. (1 mark)

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| --- |
| **Solution** |
| For every increase in n, $T\_{n}$ increases by ½. ORA constant difference pattern |
| **Specific behaviours** |
| ✓ States constant difference occurs |

 **(ii)** Determine the recursive definition for the

 arithmetic sequence shown, where T0 = -3,

 T2 = -2, T4 = -1, T6 = 0 and T8 = 1. (2 marks)

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| **Solution** |
| $T\_{n}=T\_{n-1}+\frac{1}{2}: T\_{0}=-3$ OR$ T\_{n+1}=T\_{n}+\frac{1}{2}: T\_{1}=-2.5$ |
| **Specific behaviours** |
| ✓ States pattern✓ States first term |

**Question 4 (7 marks)**

The scatterplots for three sets of data, $A$, $B$ and $C$ are shown below.



(a) Order the data sets, from strongest to weakest, in terms of the strength of the linear correlation coefficient between the variables. Explain your answer. (3 marks)

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| --- |
| **Solution** |
| $ACB$ $B$ must be last, as little association is evident between $x$ and $y$ - as $x$ changes, $y$ does not appear to.Points in $A$ lie closer to a straight line than $C$.*(NB* $r\_{A}≈0.9, r\_{B}≈0.4, r\_{C}≈0.7$*)* |
| **Specific behaviours** |
| ✓ order✓ reason for B last✓ reason for A first |

(b) One of the relationships appears to be non-linear. Identify this data set and explain your choice. (2 marks)

|  |
| --- |
| **Solution** |
| $A$ Points appear curved, bowing upwards. |
| **Specific behaviours** |
| ✓ chooses $A$✓ indicates curve |

(c) Give one example of a pair of possible variables for dataset $B$, explaining your choice.

 (2 marks)

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| --- |
| **Solution** |
| Examples: Day of month v weight; Age v income for adults employed in same job; and so on.As one increases, it would be reasonable to expect the other to not change. |
| **Specific behaviours** |
| ✓ clear example of two variables✓ explains as one increases other would stay the same |

Question 5 (14 marks)

Consider the recursive rule $T\_{n+1}=aT\_{n}+b, T\_{1}=32$, where $a$ and $b$ are both constants.

(a) If $a=\frac{1}{2}$ and $b=0$, determine

|  |
| --- |
| **Solution** |
| $$\frac{1}{2}×32=16,\frac{1}{2}×16=T\_{3}=8$$ |
| **Specific behaviours** |
| ✓ states value |

 (i) $T\_{3}$. (2 marks)

(ii) a rule for the $n^{th}$ term of the sequence. (2 marks)

|  |
| --- |
| **Solution** |
| $$T\_{n}=32\left(\frac{1}{2}\right)^{n-1}$$ |
| **Specific behaviours** |
| ✓ uses form of equation✓ uses correct values |

(iii) the value of $n$ if $T\_{n}=\frac{1}{4}$. (1 mark)

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| --- |
| **Solution** |
| $$32, 16, 8, 4, 2, 1,\frac{1}{2},\frac{1}{4}⇒n=8$$ |
| **Specific behaviours** |
| ✓ states value |

(b) If $a=1$ and $b=4$, determine

(i) $T\_{2}$, $T\_{3}$ and $T\_{4}$. (2 marks)

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| --- |
| **Solution** |
| $$T\_{2}=32+4=36, T\_{3}=36+4=40, T\_{4}=40+4=44$$ |
| **Specific behaviours** |
| ✓ determines $T\_{2}$✓ determines all terms |

(ii) a rule for the $n^{th}$ term of the sequence. (2 marks)

|  |
| --- |
| **Solution** |
| $$T\_{n}=32+(n-1)(4)$$ |
| **Specific behaviours** |
| ✓ uses correct form of equation✓ uses correct values |

(iii) $T\_{101}$. (1 mark)

|  |
| --- |
| **Solution** |
| $$T\_{101}=32+100×4=432$$ |
| **Specific behaviours** |
| ✓ correct value |

(c) If $a=\frac{1}{2}$ and $b=4$, determine

(i) $T\_{2}$ and $T\_{5}$. (2 marks)

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| --- |
| **Solution** |
| $$T\_{2}=\frac{1}{2}\left(32\right)+4=20$$$$T\_{3}=14, T\_{4}=11, T\_{5}=9.5$$ |
| **Specific behaviours** |
| ✓ determines $T\_{2}$✓ determines $T\_{5}$ |

(ii) the value $T\_{n}$ approaches as $n$ becomes very large. Justify your answer. (2 marks)

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| --- |
| **Solution** |
| $$x=\frac{1}{2}x+4$$$$\frac{1}{2}x=4⇒x=8$$$$T\_{n} will approach 8$$ |
| **Specific behaviours** |
| ✓ indicates equation for steady state✓ solves equation |

Question 6 (4 marks)

Use the table below to match each recursive equation with the statement.

Use the letter of each recursive equation (A, B, C, D) and the letter of the statement (a, b, c, d).

A:  B: 

C:  D: 

a: Each term is obtained by the previous term being multiplied by 3 then divided by 2.

b: Each term is obtained by doubling the previous term, then dividing by 3.

c: Each term is obtained by adding 3 to the previous term, then multiplying by 2.

d: Each term is obtained by the previous term being added to 2, then multiplied by 1.5.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Recursive****Equation** | **A** | **B** | **C** | **D** |
| **Statement** | **C** | **B** | **A** | **D** |

Question 7 (6 marks)

The scatterplot below illustrates how the number of firefighters sent to fires ($x$) is related to the cost of the damage ($y$) resulting from the fire, in a large city.



(a) Place a cross on the scale below for the best estimate of the correlation coefficient between the two variables. Explain your choice. (2 marks)



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| --- |
| **Solution** |
| See line. A fairly strong, positive relationship is shown. |
| **Specific behaviours** |
| ✓ indicates value between 0.6 and 0.9✓ comment indicates strength and direction |

(b) Is it reasonable to conclude from the scatterplot that sending fewer firefighters to a fire will decrease the cost of the damage? Explain your answer. (2 marks)

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| --- |
| **Solution** |
| No.An observed association does not mean a causal relationship exists. |
| **Specific behaviours** |
| ✓ indicates NO✓ explains no causal relationship |

(c) Identify and explain a possible non-causal explanation for the observed association between the number of firefighters and the cost of the damage. (2 marks)

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| --- |
| **Solution** |
| The size of the fire.A larger fire is likely to cause greater damage and require more fireman to put it out. |
| **Specific behaviours** |
| ✓ identifies third variable✓ explains likely cause of association |

Question 8 (6 marks)

A set of data, (two variables *P* and *t*) has a line of regression equation  and a coefficient of determination of 0.64.

**(a)** State whether the following statements are true (T) or false (F). (4 marks)

**(i)** *P* is the response variable. \_\_T\_\_\_\_\_

**(ii)** As *t* increases, *P* increases. \_\_F\_\_\_\_\_

**(iii)** An outlier would increase the correlation coefficient. \_\_F\_\_\_\_\_

 **(iv)** For a linear regression model to be used to predict future values,

 the graph of the residuals will show a random pattern. \_\_T\_\_\_\_\_

|  |
| --- |
| **Solution** |
| 1. True
2. False
3. False
4. True
 |
| **Specific behaviours** |
| ✓ One mark for each correct answer |

**(b)** If the residual is -0.2 when *t* = 5, state the real value of *P.* (2 marks)

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| --- |
| **Solution** |
| $$P=-0.15\left(5\right)+2.2$$$$P=1.45$$$$-0.2=x-1.45$$$$Observed=1.25$$ |
| **Specific behaviours** |
| ✓ Calculates the predicted value✓ Calculates the real value of P |

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

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Question number: \_\_\_\_\_\_\_\_\_

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